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Astronomical Algorithms for use with Micro-computers

by

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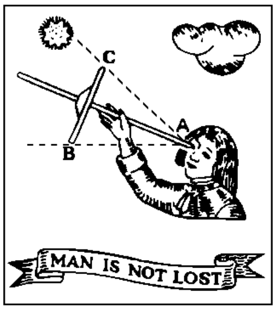
Summary

Long-term low-precision astronomical algorithms required in astronomy and astro-navigation designed especially for micro-computers are given.



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Typeset using TEX.

1. **Conversion from Calendar Date to Julian Date**

In section 1 and 2 the following mathematical notation is used

$$[x] \quad \text{means take the integer part of } x$$

The function [ ] has been adopted because most calculators have a function key usually called **int** which performs the calculation and most high level computer languages also have this function. The use of MOD and DIV has deliberately been avoided because problems may arise in their use, *e.g.* in BASIC the MOD function assumes integer arguments.

The Julian date JD at $0^{\rm h}$ UT (*i.e.* midnight) on calendar date Y–M–D where Y is the year, M is the month and D is the day of the month is calculated as follows:

Step 1. If $M > 2$ set $y = Y$ and $m = M - 3$, otherwise set $y = Y - 1$ and $m = M + 9$

Step 2. Calculate

$$J = [365{\cdot}25(y + 4712)] + [30{\cdot}6m + 0{\cdot}5] + 59 + D - 0{\cdot}5$$

Step 3. If the calendar date is Gregorian set

$$G = 38 - [3\,[49 + y/100]/4]$$

For the Julian calendar $G = 0$

Step 4. Calculate the Julian date from

$$JD = J + G$$

This method is based on a paper by D.A. Hatcher, Quarterly Journal of the Royal Astronomical Society, 1984, **25**, 53 who also gives the inverse algorithm Julian date to Civil Date.

The day of the week is obtained from $I = JD - 7[JD/7]$ with $I = 0$ corresponding to Monday *etc.*

Assuming a mean moon, and a Julian date JD the new moon occurs within about one day of

$$\text{Julian date} = ([JD/29{\cdot}530588] + 0{\cdot}33)\,29{\cdot}530588$$

Worked Example. Find the Julian date (JD) for the Gregorian calendar date 1990 March 5. $Y = 1990$, $M = 3$, $D = 5$.

Step 1. $y = 1990$ $m = 0$

Step 2. $J = 2447968{\cdot}5$

Step 3. $G = -13$

Step 4. $JD = 2447955{\cdot}5$

2. **Conversion from Julian Date to Calendar Date**

The calender date Y–M–D where Y is the year, M is the month and D is the day of the month on Julian date JD at $0^{\rm h}$ UT is calculated as follows:

Step 1. Calculate the Julian day number JDN from

$$JDN = JD + 0{\cdot}5$$

Step 2. If the calendar date is Gregorian set

$$G = [3\,[\,(JDN - 4479{\cdot}5)/36524{\cdot}25]/4 + 0{\cdot}5] - 37$$

For the Julian calendar $G = 0$

Step 3. Calculate the quantities

$$J1 = JDN + G$$
$$J2 = J1 - 59{\cdot}25$$
$$J3 = [J2 - 365{\cdot}25\,[J2/365{\cdot}25]\,]$$
$$J4 = J3 + 0{\cdot}5$$
$$J5 = [J4/30{\cdot}6] + 2$$

Step 4. Calculate Y, M, D from

$$Y = [J1/365{\cdot}25] - 4712$$
$$M = J5 - 12\,[J5/12] + 1$$
$$D = [J4 - 30{\cdot}6\,[J4/30{\cdot}6]\,] + 1$$

Worked Example. Find the Gregorian calendar date Y–M–D for the Julian date (*JD*) 2447955·5.

Step 1.	$JDN = 2447956{\cdot}0$				
Step 2.	$G = 13$				
Step 3.	$J1 = 2447969$	$J2 = 2447909{\cdot}75$	$J3 = 4$	$J4 = 4{\cdot}5$	$J5 = 2$
Step 4.	$Y = 1990$	$M = 3$ (March)	$D = 5$		

3. **Apparent Solar Ephemeris for the period BC 2000 – AD 2200**

This algorithm calculates the apparent right ascension, declination and semi-diameter of the Sun to an accuracy of $0'\!.6$ over the period BC 2000 – AD 2200.

Method

Step 1. Convert civil date Y–M–D to Julian date *JD* at $0^{\rm h}$ UT.

Calculate $D = JD + h/24 - 2451545{\cdot}0$

and $t = D/36525$

where t is the interval from 2000 January 1 at $12^{\rm h}$ UT in Julian centuries of 36525 days.

Step 2. Calculate the corresponding time interval T in a uniform time scale (Dynamical Time) from

$$T = t + \Delta T(t)$$

where $\Delta T(t) = (28{\cdot}43 + 4{\cdot}525t + 1{\cdot}404t^2) \times 10^{-8}$ between 390 BC to 948 AD

$\Delta T(t) = 0{\cdot}808(t-2)^2 \times 10^{-8}$ elsewhere

This is based on formulae by Stephenson and Morrison in Phil. Trans. R. Soc. Lond. **A 313**, 47–70, (1984), "Long-term changes in the rotation of the Earth: 700 BC to AD 1980". Current values are published in section K of *The Astronomical Almanac.*

Step 3. Calculate the solar arguments; the geometric mean ecliptic longitude of date (λ_m), the mean anomaly (G), the mean obliquity of the ecliptic (ε_0), and the equation of the centre (C).

$$\lambda_m = 280^\circ\!.466\,45 + 36000^\circ\!.769\,75\,T + 0^\circ\!.000\,3132\,T^2 \qquad \text{remove multiples of } 360^\circ$$
$$G = 357^\circ\!.529 + 35999^\circ\!.050\,29\,T \qquad \text{remove multiples of } 360^\circ$$
$$\varepsilon_0 = 23^\circ\!.4393 - 0^\circ\!.013\,01\,T - 0^\circ\!.000\,0001\,T^2 + 0^\circ\!.000\,0006\,T^3$$
$$C = (1^\circ\!.9147 - 0^\circ\!.004\,82\,T - 0^\circ\!.000\,015\,T^2)\,\sin G + 0^\circ\!.019\,99\,\sin 2G$$

Step 4. Apply aberration, correction to the centre and nutation to obtain λ, the apparent ecliptic longitude of date. Calculate ε, the true obliquity of the ecliptic. The apparent ecliptic latitude $\beta = 0^\circ$.

Calculate $\Omega = 125^\circ\!.045 - 1934^\circ\!.136\,T$ remove multiples of 360°

$$\varepsilon = \varepsilon_0 + 0^\circ\!.0026\cos\Omega$$
$$\lambda = \lambda_m + C - 0^\circ\!.0057 - 0^\circ\!.0048\sin\Omega \qquad \text{remove multiples of } 360^\circ$$

Step 5. Convert to apparent right ascension and declination of date (α, δ) and form the semi-diameter (SD).

Calculate $x = \cos\lambda$, $y = \cos\varepsilon\sin\lambda$, $z = \sin\varepsilon\sin\lambda$

$$A = \tan^{-1} y/x = \tan^{-1}(\cos\varepsilon\tan\lambda)$$

If $x < 0$ then $\alpha = A + 180^\circ$

If $x > 0$ and $y < 0$ then $\alpha = A + 360^\circ$ otherwise $\alpha = A$

$$\delta = \sin^{-1} z = \sin^{-1}(\sin\varepsilon\sin\lambda)$$
$$SD = 0^\circ\!.2666/(1 - 0{\cdot}017\cos G)$$

Step 6. Calculate GHA from

$$GHA = 100^\circ\!.4606 + 36000^\circ\!.769\,98\,t + 0^\circ\!.000\,387\,t^2 + 15h - 0^\circ\!.0048\sin\Omega\cos\varepsilon_0 - \alpha$$

Remove multiples of 360°. In order to preserve accuracy it is advisable to separate the t term into two parts *viz.* $36000\,t + 0{\cdot}769\,98\,t$. Multiples of 360° should be remove from the first term before the second terms is added to it.

Note: 1. A one line expression for the right ascension in degrees is given by

$$\alpha = \lambda - yf\,\sin 2\lambda + 0{\cdot}5y^2 f\,\sin 4\lambda$$

where $y = \tan^2\varepsilon/2$ and $f = 180/\pi$

This is a series approximation of $\tan^{-1}(\cos\varepsilon\tan\lambda)$ and is correct to order y^2 (0′.1). The factor f converts the last two terms of the series from radians to degrees.

2. The equation of time E is calculated at step 6 from

$$E = GHA - (15h - 180^\circ)$$

If $E > 10^\circ$ then $E = E - 360^\circ$

The accuracy of this algorithm is limited by the exclusion of the planetary perturbations, which are periodic terms, the precision of the solar arguments, *i.e.* λ_m *etc.*, and the approximation to the effect of nutation. In order to obtain valid angles when T is large the functions of T have been printed to a specified precision which is indicated by the order of the polynomials and the number of decimal places of the coefficients. Users who are interested in short periods, *i.e.* $-1 \le T \le +1$, may ignore all T^3 and T^2 terms as well as the T coefficient from the $\sin 2G$ term in the equation of the centre (C), with only a slight loss of accuracy. Users interested in periods outside those mentioned in this paper should use the definitive arguments which are obtainable in the supplement to the 1984 *Astronomical Almanac*.

The accuracy is further limited by the accuracy of ΔT. An error of 2^{m} in ΔT produces an error of 0′.1 in right ascension. If ΔT is taken to be 1^{m} this will cover the period 1600 to 2100 with little loss of accuracy.

Errors in the method

The errors to be expected when using this method to calculate the right ascension and declination of the Sun have been investigated. The right ascensions and declinations were calculated daily from the beginning of the start year for a period of over 20 years. For the comparison Newcomb's algorithm, with the equinox shifted onto the FK5 system was used. A direct comparison with the FK5 Sun was also made for the period starting 1980 and the errors were the same in both cases. The maximum errors for various periods are shown below.

Maximum Absolute Errors

Years	−2000	0	1900	2100	2200
α	$\pm 0'.6$	$\pm 0'.6$	$\pm 0'.5$	$\pm 0'.5$	$\pm 0'.5$
δ	$\pm 0'.5$	$\pm 0'.3$	$\pm 0'.2$	$\pm 0'.2$	$\pm 0'.2$

The accuracy of *GHA* is similar to α but it will be further degraded by the term $15h$, for example an error in h of $0^{s}.5$ produces an error of $0'.125$ in *GHA*. The accuracy of the semi-diameter (*SD*) is better than $0'.1$.

Worked Example. Find the *GHA*, *Dec*, *SD* and equation of time, *E*, of the Sun on 1978 January 3 at $7^{h}\ 30^{m}$ UT.

Step 1. $JD = 2443511{\cdot}5 + 7{\cdot}5/24$ $\quad D = -8033{\cdot}1875$ $\quad t = -0{\cdot}219\,936\,687$

Step 2. $\Delta T(t) = 84^{s}/(3600 \times 24 \times 36525)$ $\quad T = -0{\cdot}219\,936\,661$

Step 3. $\lambda_m = 282{\cdot}576\,99$ $\quad G = 0{\cdot}0171$ $\quad \varepsilon_0 = 23{\cdot}44216$
$C = 0{\cdot}000\,583$

Step 4. $\Omega = 190{\cdot}432\,4$ $\quad \varepsilon = 23{\cdot}439\,60$
$\lambda = 282{\cdot}572\,74$ $\quad \beta = +0{\cdot}0$

Step 5. $x = +0{\cdot}217\,679$ $\quad y = -0{\cdot}895\,479$ $\quad z = -0{\cdot}388\,243$
$\alpha = 283^{\circ}.6628$ $\quad \delta = -22^{\circ}.8452$ $\quad SD = 0^{\circ}.2716$

Step 6. $GHA = 291^{\circ}.4086$ $\quad E = -1^{\circ}.0914$

4. Apparent Places of Navigational Stars

This algorithm calculates the Apparent Places of the Navigational Stars using ecliptic longitude, latitude and their centennial rates of change for equinox and epoch J2000·0.

Notation

λ_0, β_0 = ecliptic longitude and latitude for epoch and equinox J2000·0
μ, μ' = centennial rates of change of ecliptic longitude and latitude

Method

Step 1. Convert civil date *Y–M–D* to Julian date *JD*. If apparent place is needed at UT = h then add $h/24$ to *JD*.

Calculate $D = JD - 2451545{\cdot}0$

and $t = D/36525$

which is the interval from 2000 January 1 at 12^{h} UT in Julian centuries. Strictly speaking as in step 2 of section 3 the time argument should be $T = t + \Delta T(t)$ in all steps except when calculating *GHA* Aries in step 7, which is a function of UT. However since the movement of the stars is very small in the time interval ΔT the change in time scales produces negligible differences.

Step 2. Calculate the ecliptic longitude and latitude for epoch of date and equinox J2000·0 from

$$\lambda_1 = \lambda_0 + \mu\, t$$
$$\beta_1 = \beta_0 + \mu'\, t$$

Longitude and Latitude referred to the ecliptic and mean equinox of J2000·0

No.	Name			λ_0	μ	β_0	μ'
				°	°	°	°
1	Acamar	θ	Eri	23·2723	−0·00152	− 53·7402	+0·00112
2	Achernar	α	Eri	345·3117	+0·00285	− 59·3783	−0·00275
3	Acrux	α	Cru	221·8701	−0·00047	− 52·8787	−0·00070
4	Adhara	ε	CMa	110·7630	+0·00025	− 51·3602	+0·00010
5	Aldebaran	α	Tau	69·7892	+0·00104	− 5·4674	−0·00550
6	Alioth	ε	UMa	158·9334	+0·00417	+ 54·3188	+0·00194
7	Alkaid	η	UMa	176·9331	−0·00430	+ 54·3880	−0·00230
8	Al Na'ir	α	Gru	315·9070	+0·00184	− 32·9133	−0·00536
9	Alnilam	ε	Ori	83·4636	−0·00002	− 24·5064	−0·00007
10	Alphard	α	Hya	147·2792	−0·00074	− 22·3825	+0·00067
11	Alphecca	α	CrB	222·2959	+0·00568	+ 44·3236	−0·00118
12	Alpheratz	α	And	14·3085	+0·00162	+ 25·6804	−0·00575
13	Altair	α	Aql	301·7765	+0·01939	+ 29·3035	+0·00733
14	Ankaa	α	Phe	345·4938	−0·00100	− 40·6331	−0·01237
15	Antares	α	Sco	249·7623	−0·00007	− 4·5699	−0·00061
16	Arcturus	α	Boo	204·2337	−0·00768	+ 30·7363	−0·06288
17	Atria	α	TrA	260·8962	+0·00123	− 46·1513	−0·00075
18	Avior	ε	Car	173·1294	−0·00250	− 72·6798	−0·00013
19	Bellatrix	γ	Ori	80·9464	−0·00032	− 16·8161	−0·00037
20	Betelgeuse	α	Ori	88·7547	+0·00080	− 16·0270	+0·00026
21	Canopus	α	Car	104·9614	+0·00308	− 75·8239	+0·00076
22	Capella	α	Aur	81·8579	+0·00126	+ 22·8643	−0·01191
23	Deneb	α	Cyg	335·3293	+0·00029	+ 59·9061	−0·00002
24	Denebola	β	Leo	171·6176	−0·01153	+ 12·2669	−0·00849
25	Diphda	β	Cet	2·5835	+0·00673	− 20·7836	−0·00191
26	Dubhe	α	UMa	135·1975	−0·00239	+ 49·6802	−0·00343
27	Elnath	β	Tau	82·5750	+0·00037	+ 5·3851	−0·00491
28	Eltanin	γ	Dra	267·9687	−0·00080	+ 74·9223	−0·00055
29	Enif	ε	Peg	331·8850	+0·00090	+ 22·0999	−0·00029
30	Fomalhaut	α	PsA	333·8604	+0·00716	− 21·1357	−0·00802
31	Gacrux	γ	Cru	216·7397	+0·00737	− 47·8312	−0·00543
32	Gienah	γ	Crv	190·7256	−0·00449	− 14·5009	−0·00128
33	Hadar	β	Cen	233·7925	−0·00036	− 44·1375	−0·00076
34	Hamal	α	Ari	37·6625	+0·00364	+ 9·9651	−0·00569
35	Kaus Australis	ε	Sgr	275·0787	−0·00106	− 11·0519	−0·00346
36	Kochab	β	UMi	133·3195	−0·00112	+ 72·9876	−0·00088
37	Markab	α	Peg	353·4857	+0·00125	+ 19·4060	−0·00182
38	Menkar	α	Cet	44·3201	−0·00091	− 12·5856	−0·00197
39	Menkent	θ	Cen	222·3086	−0·00873	− 22·0800	−0·01871
40	Miaplacidus	β	Car	211·9692	−0·01254	− 72·2357	−0·00329
41	Mirfak	α	Per	62·0810	+0·00051	+ 30·1255	−0·00084
42	Nunki	σ	Sgr	282·3853	+0·00026	− 3·4495	−0·00156
43	Peacock	α	Pav	293·8176	−0·00041	− 36·2677	−0·00244
44	Pollux	β	Gem	113·2156	−0·01700	+ 6·6842	−0·00436
45	Procyon	α	CMi	115·7855	−0·01504	− 16·0196	−0·03143
46	Rasalhague	α	Oph	262·4487	+0·00459	+ 35·8352	−0·00609
47	Regulus	α	Leo	149·8292	−0·00648	+ 0·4649	−0·00222
48	Rigel	β	Ori	76·8295	−0·00003	− 31·1228	−0·00007
49	Rigil Kentaurus	α	Cen	239·4793	−0·13521	− 42·5959	−0·02399
50	Sabik	η	Oph	257·9696	+0·00084	+ 7·1978	+0·00275
51	Schedar	α	Cas	37·7838	+0·00105	+ 46·6222	−0·00157
52	Shaula	λ	Sco	264·5858	+0·00007	− 13·7884	−0·00079
53	Sirius	α	CMa	104·0816	−0·01524	− 39·6053	−0·03492
54	Spica	α	Vir	203·8414	−0·00075	− 2·0545	−0·00118
55	Suhail	λ	Vel	161·1877	−0·00116	− 55·8708	+0·00011
56	Vega	α	Lyr	285·3164	+0·01403	+ 61·7328	+0·00709
57	Zubenelgenubi	α^2	Lib	225·0827	−0·00226	+ 0·3330	−0·00267
58	Polaris	α	UMi	88·5676	+0·00098	+ 66·1014	−0·00118
59	σ Octantis	σ	Oct	271·8706	+0·00118	− 65·8402	−0·00042

Step 3. Apply aberration from

$$\lambda_\odot = 280^\circ\!.460 + 36000^\circ\!.770\,t \qquad \text{remove multiples of } 360^\circ$$

$$\lambda_2 = \lambda_1 - 0^\circ\!.0057\cos(\lambda_1 - \lambda_\odot)/\cos\beta_1$$

$$\beta_2 = \beta_1 + 0^\circ\!.0057\sin(\lambda_1 - \lambda_\odot)\sin\beta_1$$

Step 4. Apply precession from J2000·0 to epoch of date.

Calculate

$$a = 1^\circ\!.39697\,t + 0^\circ\!.000309\,t^2$$

$$b = 0^\circ\!.0131\,t - 0^\circ\!.00001\,t^2$$

$$c = 5^\circ\!.1236 + 0^\circ\!.2416\,t$$

$$\beta_3 = \beta_2 + b\sin(\lambda_2 + c)$$

$$\lambda_3 = \lambda_2 + a - b\cos(\lambda_2 + c)\tan\beta_3$$

Step 5. Apply nutation to obtain the apparent ecliptic longitude and latitude (λ, β) of date.

Calculate

$$\Omega = 125^\circ\!.045 - 1934^\circ\!.136\,t \qquad \text{remove multiples of } 360^\circ$$

$$\varepsilon_0 = 23^\circ\!.4393 - 0^\circ\!.0130\,t$$

$$\varepsilon = \varepsilon_0 + 0^\circ\!.0026\cos\Omega$$

$$\lambda = \lambda_3 - 0^\circ\!.0048\sin\Omega$$

$$\beta = \beta_3$$

Step 6. Convert to apparent right ascension and declination of date $(\alpha,\ \delta)$.

Calculate

$$x = \cos\beta\cos\lambda$$

$$y = \cos\varepsilon\cos\beta\sin\lambda - \sin\varepsilon\sin\beta$$

$$z = \sin\varepsilon\cos\beta\sin\lambda + \cos\varepsilon\sin\beta$$

$$A = \tan^{-1} y/x$$

If $x < 0$ then $\alpha = A + 180^\circ$

If $x > 0$ and $y < 0$ then $\alpha = A + 360^\circ$ otherwise $\alpha = A$

$$\delta = \sin^{-1} z$$

Step 7. Calculate *GHA* from

$$GHA = 100^\circ\!.4606 + 36000^\circ\!.77005\,t + 0^\circ\!.000388\,t^2 + 15h - 0^\circ\!.0048\sin\Omega\cos\varepsilon - \alpha$$

Remove multiples of 360°. In order to preserve accuracy it is advisable to separate the t term into two parts *viz.* $36000\,t + 0{\cdot}77005\,t$. Multiples of 360° should be remove from the first term before the second terms is added to it.

Errors in the method

The errors to be expected when using this method to calculate the right ascension and declination of the navigational stars have been investigated. Three stars *Altair*, *Rigil Kentaurus*, *Sirius* have large space motions and over long periods of time (more than 200 years) they require further corrections of order t^2. The corrections $\mu_1 t^2$ and $\mu'_1 t^2$ which are added to λ and β, respectively are given in the table.

	Altair	*Rigil Kentaurus*	*Sirius*
μ_1	$+0^\circ\!.000\,012$	$-0^\circ\!.000\,283$	$-0^\circ\!.000\,012$
μ'_1	$+0^\circ\!.000\,003$	$+0^\circ\!.000\,039$	$-0^\circ\!.000\,009$

The maximum errors for all the navigational stars for various periods which were found by comparing right ascension and declinations every 50 days are as follows:

2000 ± 2000 years		From 1000 to 1800		2000 ± 200 years	
years	max error	years	max error	years	max error
0 – 200	0′.4	1000 – 1100	8″	1950 – 2050	2″.6
200 – 400	0′.3	1100 – 1200	7″	1800 – 2200	3″
400 – 900	0′.2	1200 – 1400	6″		
900 – 3000	0′.1	1400 – 1600	5″		
3000 – 3500	0′.2	1600 – 1800	4″		
3500 – 3800	0′.3				
3800 – 4000	0′.4				

The accuracy of *GHA* is similar to α but it is further degraded by the term $15h$, for example an error in h of $0^s.5$ produces an error of $0'.125$ in *GHA*.

Worked Example. Find the *GHA* and *Dec* of *Vega* on 1978 January 3 at $7^h\ 30^m$ UT.

From the table $\lambda_0 = 285{\cdot}3164$, $\mu = +0{\cdot}01403$, $\beta_0 = +61{\cdot}7328$, $\mu' = +0{\cdot}00709$.

Step 1.	$JD = 2443511{\cdot}5 + 7{\cdot}5/24$	$D = -8033{\cdot}1875$	$t = -0{\cdot}219\,936\,687$
Step 2.	$\lambda_1 = 285{\cdot}313\,314$	$\beta_1 = +61{\cdot}731\,241$	
Step 3.	$\lambda_\odot = 282{\cdot}569\,91$		
	$\lambda_2 = 285{\cdot}301\,293$	$\beta_2 = +61{\cdot}731\,481$	
Step 4.	$a = -0{\cdot}307\,230\,007$	$b = -0{\cdot}002\,881\,654$	$c = 5{\cdot}070\,4633$
	$\lambda_3 = 284{\cdot}995\,928$	$\beta_3 = +61{\cdot}734\,182$	
Step 5.	$\Omega = 190{\cdot}432\,46$	$\varepsilon = 23{\cdot}439\,602$	
	$\lambda = 284{\cdot}996\,798$	$\beta = +61{\cdot}734\,182$	
Step 6.	$x = +0{\cdot}122\,541\,55$	$y = -0{\cdot}770\,036\,64$	$z = +0{\cdot}626\,120\,81$
	$\alpha = 279{\cdot}042\,084$	$\delta = +38{\cdot}764\,500$	
Step 7.	$GHA = 296{\cdot}0292$	$\delta = +38{\cdot}7645$	