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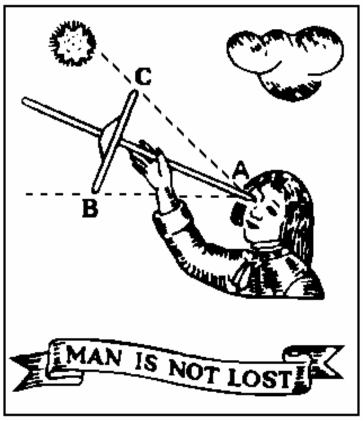
No. 47

Approximate Solar Coordinates

by

B. Emerson



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Summary

This note gives an algorithm for calculating the coordinates of the Sun to a precision of about $0\overset{\circ}{.}001$ in angle, 0.00002 a.u. in distance and 0.2 km/s in velocity.

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Approximate solar coordinates

The purpose of this note is to give an algorithm for the calculation of the coordinates of the Sun to a precision of about $0^\circ.001$ in angle, 0.00002 a.u. in distance and 0.1 km/s in velocity. For dates several thousand years ago the precision will be an order of magnitude less than these values. The basic quantities are the geometric ecliptic longitude (λ_m) referred to the mean equinox of date (m.e.d.) and the radius vector (R); the ecliptic latitude (m.e.d.) is zero to this precision. These quantities are calculated from two series whose terms have been taken from Newcomb's theory of the motion of the Earth around the Sun (Tables of the Sun, Astronomical Papers of the American Ephemeris, 6, part 1, 1895); many small terms have been omitted and the angular quantities have been expressed in degrees and decimals for ease of use in small calculators. Formulae are also given for the derivation of the Sun's apparent longitude, right ascension, declination, Greenwich hour angle, equatorial rectangular coordinates and geocentric velocity components. An example is included which shows the value of all the terms used in the algorithm.

Time scales

This algorithm enables results to be calculated for a given universal time (U.T.), the starting data being

$$JD = \text{Julian date (.5) at } 0^h \text{ U.T.}$$
$$h = \text{U.T. time of day in days}$$

Due to the variable rate of rotation of the Earth U.T. is a non-uniform time scale and is not the time argument used in dynamical theories of planets. Instead a dynamical time called ephemeris time (E.T.) is used, which is related to U.T. by means of the equation E.T. = U.T. + Δt. The following expression for Δt, in Julian centuries, is based upon a formula by P.M. Muller ("An analysis of the ancient astronomical observations with the implications for geophysics and cosmology", D.Ph. Thesis, 1975, Newcastle University)

$$\Delta t = \left\{-3.36 + 1.353\ (t + 1.33)^2\right\} \times 10^{-8}$$

where

$$t = (JD + h - 2415020.0)/36525.0$$

which is the number of days from 1900 January 0.5 expressed in Julian centuries. To the precision of this algorithm Δt is negligible during the approximate period 1650 A.D. to 1950 A.D. Before 1650 A.D. Δt must be used; for current dates Δt from the above quadratic is sufficiently close to the actual values, and should remain so up to about the year 2000 A.D. The time variable needed to evaluate the algorithm can now be formed from

$$T = t + \Delta t$$

In this algorithm it is sufficient to have four decimal places in h.

Longitude

(a) Quantities which are needed in the expression for the Sun's geometric longitude (λ_m) referred to the mean equinox of date are: the mean anomalies of Venus (M_V), Earth (M_E), Mars (M_M), Jupiter (M_J); and the mean elongation of the Moon from the Sun (D). They are:

$$M_V = 212.6 + 58517.80\ T$$
$$M_E = 358.476 + 35999.0498\ T$$
$$M_M = 319.5 + 19139.86\ T$$
$$M_J = 225.3 + 3034.69\ T$$
$$D = 350.7 + 445267.11\ T$$

M_E has more decimal places than the other angles because it is combined with a larger coefficient in the following eccentricity term and T needs eight decimals for the linear term in λ_m.

Hence λ_m may be written

$$
\begin{aligned}
\lambda_m = \; & 279.69668 + 36000.768925\ T + 0.0003025\ T^2 \\
& +(1.91946 - 0.004789\ T) \sin M_E + 0.02009 \sin 2M_E + 0.00029 \sin 3M_E \\
& + 0.00154 \cos (148.3 + 2M_V - 2M_E) \\
& + 0.00134 \cos (299.1 + M_V - M_E) \\
& + 0.00069 \cos (315.9 + 2M_V - 3M_E) \\
& + 0.00043 \cos (345.3 + 3M_V - 4M_E) \\
& + 0.00028 \cos (318.2 + 3M_V - 5M_E) \\
& + 0.00057 \cos (343.9 - 2M_M + 2M_E) \\
& + 0.00049 \cos (200.4 - 2M_M + M_E) \\
& + 0.00200 \cos (179.5 - M_J + M_E) \\
& + 0.00076 \cos (\ 87.1 - 2M_J + 2M_E) \\
& + 0.00072 \cos (263.2 - M_J) \\
& + 0.00045 \cos (109.5 - 2M_J + M_E) \\
& + 0.00179 \sin (D) \\
& + 0.00178 \sin (231.2 + 20.20\ T) \\
& + 0.00052 \sin (\ 57.2 + 150.27\ T)
\end{aligned}
$$

where the first line is the mean longitude; the second line allows for the eccentricity of the orbit; the terms containing M_V, M_M, M_J and D are the perturbations due to Venus, Mars, Jupiter and the Moon respectively; and the last two lines are long-period planetary perturbation terms.

(b) To derive the apparent longitude of the Sun (λ_a) referred to the true equinox of date two extra terms are needed, which are the constant value of aberration and the nutation in longitude. The latter depends on the longitude of the Moon's mean node (Ω) which is

$$\Omega = 259.2 - 1934.14\ T$$

Thus the apparent longitude is given by

$$\lambda_a = \lambda_m - 0.00569 - 0.00479 \sin \Omega$$

Right ascension and declination

Apparent right ascension (α) and declination (δ) of the Sun referred to the true equinox of date can be formed if the true obliquity (ε_t) is known, i.e.

$$\varepsilon_t = 23.4523 - 0.01301\ T - 0.000002\ T^2 + 0.0000005\ T^3 + 0.0026 \cos \Omega$$

Then

$$\tan \alpha = \sin \lambda_a \cos \varepsilon_t / \cos \lambda_a$$
$$\sin \delta = \sin \lambda_a \sin \varepsilon_t$$

T^2 and T^3 terms which have very small coefficients have been included because of their significance for megalithic dates, but for other dates where their contribution is obviously insignificant they may be neglected.

Greenwich hour angle (GHA)

From the quantities which have already been derived the GHA of the Sun can be computed from

$$GHA = UT + 99.6910 + 36000.76892\ t + 0.000387\ t^2 - \alpha - 0.0044 \sin \Omega$$

where UT and α are needed in degrees to four decimals. The calculation of the hour angle needs a more precise UT than the rest of this algorithm.

Radius vector

The radius vector of the Sun (R) is given by

$$\begin{aligned} R = {} & 1.000141 - (0.016748 - 0.0000418\ T) \cos M_E - 0.000140 \cos 2M_E \\ & + 0.000016 \cos (58.3 + 2M_V - 2M_E) \\ & + 0.000005 \cos (209.1 + M_V - M_E) \\ & + 0.000005 \cos (253.8 - 2M_M + 2M_E) \\ & + 0.000016 \cos (\ 89.5 - M_J + M_E) \\ & + 0.000009 \cos (357.1 - 2M_J + 2M_E) \\ & + 0.000031 \cos (D) \end{aligned}$$

where the first line allows for the eccentricity of the orbit and the terms containing M_V, M_M, M_J and D are the perturbations due to Venus, Mars, Jupiter and the Moon respectively.

Semi-diameter and horizontal parallax

The Sun's semi-diameter (S) is given by

$$S = 0.267/(1.0 - 0.017 \cos M_E)$$

where the denominator is a simplified expression for R. The Sun's horizontal parallax is constant to the precision of this note and has the value 0.002.

Rectangular coordinates

For some applications the Sun's rectangular equatorial coordinates (X, Y, Z) referred to the 1950.0 equator and equinox are required. It is necessary in this case to form the longitude and latitude in the 1950.0 reference frame by means of

$$\lambda_{50} = \lambda_m + 0.69808 - 1.396011\ T - 0.0003083\ T^2$$

$$\beta_{50} = (0.00654 - 0.013086\ T + 0.0000097\ T^2)\ \sin\ (\lambda_m + 6.2 - 1.15\ T)$$

so that the rectangular coordinates are

$$X = R\ \cos\ \lambda_{50}$$

$$Y = R\ (0.917437\ \sin\ \lambda_{50} - 0.40\ \sin\ \beta_{50})$$

$$Z = R\ (0.397881\ \sin\ \lambda_{50} + 0.92\ \sin\ \beta_{50})$$

These coordinates are precise enough to be used in applying annual parallax corrections to star places.

Velocity components

To a precision of 0.1 km/s perturbations are negligible, so that the following geocentric 1950.0 equatorial velocity components of the Sun in km/s correspond to an unperturbed elliptic orbit

$$\dot{X} = -\ 29.79\ \sin\ \lambda_{50} + 0.49$$

$$\dot{Y} = +\ 27.33\ \cos\ \lambda_{50} + 0.09$$

$$\dot{Z} = +\ 11.85\ \cos\ \lambda_{50} + 0.04$$

If λ_{50} is not already available from this algorithm the following simplified expression can be used

$$\lambda_{50} = 280.39 + 35999.373\ T + 1.92\ \sin\ (358 + 35999.0\ T)$$

Changing the signs of these components gives the heliocentric velocity of the Earth which can be used to calculate approximate stellar aberration.

Calculation of Sun coordinates on 1975 June 19 at $7^{h}\,40^{m}$ U.T.

J.D.		2442582.5
h		0.3194
t		+ 0.75462887
Δt		+ 0.00000003
T		+ 0.75462889
M_V		91.8
M_E		164.399
M_M		3.0
M_J		355.4
D		122.1
Mean longitude		86.91714
Eccentricity terms		+ 0.50504
Venus perturbations	(1)	+ 0.00154
	(2)	- 0.00092
	(3)	+ 0.00069
	(4)	+ 0.00034
	(5)	- 0.00019
Mars perturbations	(1)	+ 0.00034
	(2)	+ 0.00049
Jupiter perturbations	(1)	+ 0.00196
	(2)	+ 0.00032
	(3)	- 0.00003
	(4)	+ 0.00010
Moon perturbation		+ 0.00152
Long-period terms	(1)	- 0.00163
	(2)	+ 0.00008
λ_m		87.42679
		$= 87^{\circ}\,25'\,36''$
Ω		239.6
λ_a		87.42523
		$= 87^{\circ}\,25'\,31''$

ε_t		23.441
α		87.19397
		$= 5^{h}.812931$
		$= 5^{h}48^{m}46^{s}.6$
δ		+23.41592
		$=23^{\circ}\,24'57''$
U.T.		115.0000
GHA		294.7206
		$=294^{\circ}\,43'.2$
R unperturbed		1.016122
Venus perturbations	(1)	+ 0.000001
	(2)	- 0.000004
Mars perturbation		- 0.000004
Jupiter perturbations	(1)	- 0.000003
	(2)	+ 0.000008
Moon perturbation		- 0.000016
R		1.016104
S		0.2627
		$=15'.8$
λ_{50}		87.07122
β_{50}		- 0.00333
X		+ 0.05192
Y		+ 0.93102
Z		+ 0.40371
$\dot{X}$		-29.3
$\dot{Y}$		+ 1.5
$\dot{Z}$		+ 0.6

The quantities which have been derived should be useful to navigators and to astronomers for approximate calculations. Similar low-precision algorithms for the Sun, Moon and planets can be found in 'Formulae for computing astronomical data with hand-held calculators' in N.A.O. Technical Note No. 46 by B.D. Yallop, 1978. Newcomb's theory to full precision has been programmed in Fortran in 'Program for the calculation of ephemerides of the inner planets' in Publ. Obs. Univ. Bologna, 9, No. 2 by G. Mannino, L. Dall'Olio and L. D'Ascanio, 1965. More information about positional solar-system calculations can be found in 'The Explanatory Supplement to the Astronomical Ephemeris', 1974.

This note supersedes the previous undated version with the same title.